(light-colored in the figure above) and some of the paths were brown (dark in the figure). The shortest path was gray, and had length 18, as students knew from our work the previous day; the brown path had length 24. Students played in pairs, with one using the grey route and the other using the brown route. The person using the gray route could move only if he rolled an odd product, and he would move two spaces. The person using the brown route would move only if he rolled an even product, and he would also move two spaces.

The students started playing , and pretty soon I was hearing cries of "That's not fair!" from the players on the gray paths. They had considered themselves very clever to get the gray routes, and then they would end up losing. I asked them to keep track of who won — the person on the gray route or the person on the brown route. The person on the brown route almost always won. After they played the game, we did some analysis.

SUM	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

We made two tables of the outcomes of the roll of the dice — a sum table and a product table. (In the tables shown here, the even outcomes are shaded.) We looked at the probability of getting an even or odd sum versus the probability of getting an even or odd product, but had to leave the rest of the analysis for the next day. For homework the second day the students had to do a shortest path problem on a larger graph (to reinforce the first day's activity) and they had to write a paragraph explaining why the person on the brown path almost always won. The top math section also had to write a paragraph explaining how they would change the rules of *The Product Game* to make it fairer, but keeping the rule that the person on the gray path could only move on odd products and the person on the brown path could only move on even products.

PROD	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

The Five Dollar Problem

You and your brother get $5 a week for running errands for your grandmother. Your brother has a suggestion: Instead of splitting the money, why not roll a pair of dice and multiply to see who gets it? The lowest possible product is 1, the largest is 36, and their average is 18.5. So, if the product is 19 or more, he keeps the money, and if the product is 18 or less, you get the money. He's older than you, and very good at math, so you're inclined to trust his idea. Should you? Explain your answer.

Day 3

On this day we completed our analysis of the theoretical number of rolls needed to win *The Product Game* on each of the two routes.

The gray route was 18 spaces, so it took 9 moves to complete it. The probability of getting an odd product was 9/36 (as shown in the product chart) or 1/4, and so, after x rolls, we expect $(1/4)\cdot x$ of them to be odd. To figure out how large x should be in order to make 9 moves, we set up the equation $(1/4)\cdot x = 9$. Solving for x, we determined that it would take 36 rolls to win, theoretically.

We did the same for the brown route and used the equation $(3/4)\cdot x = 12$. We determined that it would take 16 rolls to win on the brown route, theoretically. This is why the brown route was so much more likely to win! [*Ed. note: It turns out that gray will win only 1 game for every 80 games that brown wins (approximately).*]

Next, we discussed how changes to the rules could change how fair, or unfair, the game was. For example, if gray moves three spaces at a time instead of two, the equation becomes $(1/4)\cdot x = 6$, giving $x = 24$. This means that gray will take 24 rolls on average, instead of 36, to complete the game. That's a little fairer! [*Ed. question: How many spaces would gray have to move at a time in order for the game to be fair?*]"

As an extension of this lesson I selected some of the paragraphs that the students had written on making the game fairer. The students worked in small groups. Each group was assigned a paragraph to analyze. They had to present their conclusions to the class. It took about two class periods for the groups to do their reports and for the class to discuss them. In addition to the group work, each student had to pick two or three of the paragraphs to analyze by himself and everyone had to solve the *Five Dollar Problem* (see insert).

The technique of having groups present solutions was one we used in the Rutgers workshop last summer. I have used it several times this year and find it a very useful classroom strategy. Next time I teach this lesson, I would probably begin Day 3 with a review of probability, and allow additional time on other days for presentations.

"Noel is infamous for trying to pull cons and doing anything humanly possible to get out of assignments. I handed him back his report without a grade. You see, the grammar and syntax were impeccable and Noel's forte is definitely not writing! As anticipated he asked me why I hadn't put a grade on his paper. I told him I didn't believe it was his, whereupon he pulls out a report, written by his cousin (several years his elder) and retorts, "Of course it's mine; here's the report my cousin gave me!" Needless to say, it was difficult keeping a straight face at that point."

– Sylvia Nomikos, LP '99